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GENERALIZED KNOT SYMMETRIC ALGEBRAS IN Z^*

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Abstract : In this paper we define Generalized knot Symmetric Algebras in Z^* and also define a new multiplication and prove associativity.
Key Words: Brauer Algebras, Signed Brauer Algebras, Knot graphs, Types of knots, knot mapping

1 INTRODUCTION:

Brauer $[Br]$ introduced algebras, known as Brauer's algebras in connection with the problem of decomposition of a tensor product representation into irreducible ones. These algebras have a basis consisting of undirected graphs.

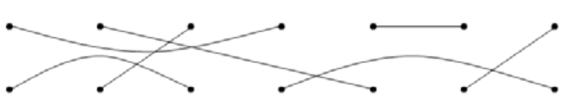
M. Parvathi and M.Kamaraj$[PK]$ introduced signed Brauer's algebra which has a basis consisting of signed diagrams. M. Kamaraj and R. Mangayarkasi[KM] introduced knot diagrams using Brauer graphs without horizontal edges. They used two types of knot only. We are motivated by the above concept replace Z^* by $\{(0,0),(1-1)^k,(-1,1)^l\}_{k,l=1}^{\infty}$

2 PRELIMINARIES

We state the basic definitions and some known results which will be used in this paper.

2.1The Brauer algebras

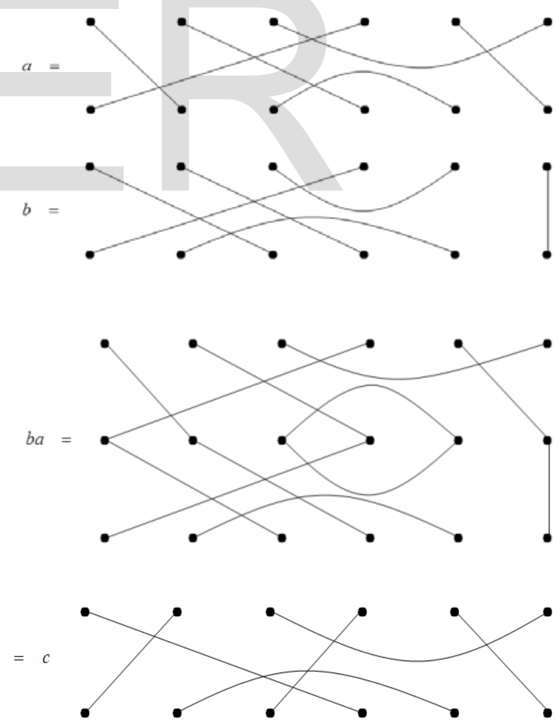
Definition [3] A Brauer graph is a graph on 2n vertices with n edges, vertices being arranged in two rows each row consisting of n vertices and every vertex is the vertex of only one edge.



2.2 Definition [3] Let V_n denote the set of Brauer graphs on 2n vertices. Let d_1, $d_2 \in V_n$. The multiplication of two graphs is defined as follows:

1. Place d_1 above d_2.
2. Join the i^{th} lower vertex of d_1 with i^{th} upper vertex of d_2.
3. Let c be the resulting graph obtain without loops. Then $ab=x^rc$, where r is the number of loops, and x is a variable.

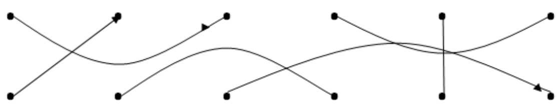
For example :



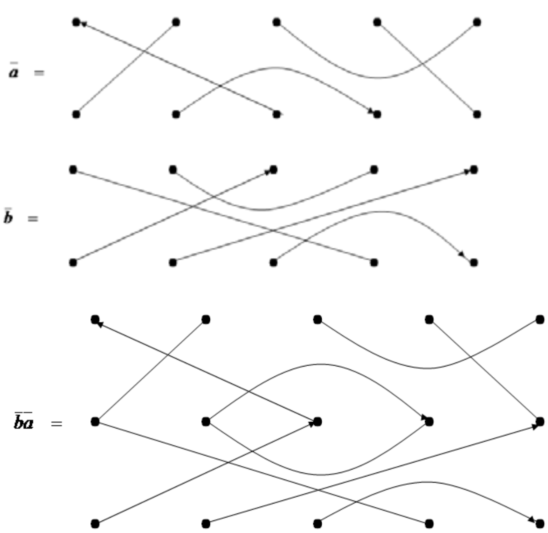
The Brauer algebra $D_n(x)$, where x is an indeterminate, is the span of the diagrams on n dots where the multiplication for the basis elements defined above. The dimension of $D_n(x)$ is $(2n)! = (2n-1)(2n-3)\ldots3.1$.

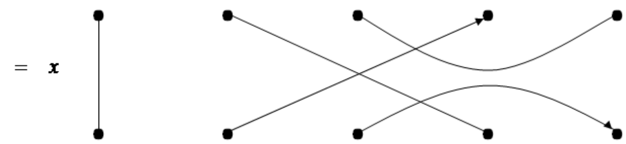
The Signed Brauer algebras

2.3 Definition [6] A signed diagram is a Brauer graph in which every edge is labeled by a + or a – sign.



2.4 Definition [6]:Let $\overline{V_n}$ denote the set of all singed Brauer graphs on 2n vertices with n singed edges.Let $\overline{D_n}(x)$ denote the linear span of $\overline{V_n}$ where x is an indeterminate. The dimension of $\overline{D_n}(x)$ is 2^n (2n)!! $=2^n$ (2n-1)(2n – 3)…3.1. Let $\overline{a},\overline{b} \in \overline{V_n}$ .Since a, b are Brauer graphs, $ab = x^d c$, the only thing we have to do is to assign a direction for every edge. An edge α in the product $\overline{a}\overline{b}$ will be labeled as a + or a- sign according as the number of negative edges involved from $\overline{a}$ and $\overline{b}$ to make α is even or odd. A loop β is said to be a positive or a negative loop in $\overline{a}\overline{b}$ according as the number of negative edges involved in the loop is even or odd. Then $\overline{a}\overline{b} = x^{2d_1+d_2}$ where d_1 is the number of positive loops and d_2 is the number of negative loops. Then is a finite dimensional algebra.





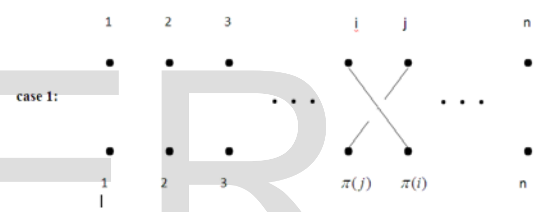
3 KNOT GRAPHS [4]

Let S_n be the symmetric group of order n and $\pi \in S_n$. A knot graph of order n is a special graph which is defined from π as follows.

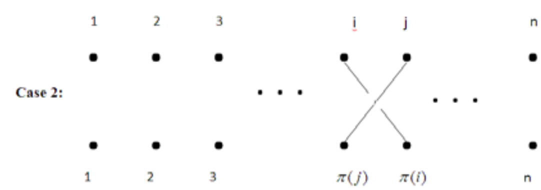
3.1 Definition

we start with an element $\pi \in S_n$, π can be represented by a graph .Consider two edges (i ,π(i)) and (j, π(j)) where vertices i and j are in the upper row and π(i)) and π(j)) are in the lower row. If i<j , $\pi(i) < \pi(j)$then edges are as in the Brauer diagram. If i<j and $\pi(j) < \pi(i)$, then we draw edges in two forms as shown below.

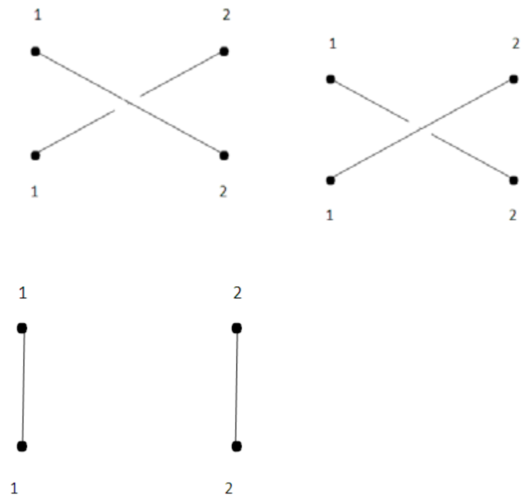
In form 1, we say (i, $\pi(i)$) is the upper edge than (j, $\pi(j)$). In this case we may also say that (j, $\pi(j)$) is lower than (i, $\pi(i)$).



In form 2, we say that (j, $\pi(j)$) is the upper edge than (i , $\pi(i)$). In this case we may also say that (i, $\pi(i)$) is lower than (j, $\pi(j)$). The above graph is called Knot graph of order n with respect to π.



Example: knot graph of order 2



4 Generalized knot Symmetric Algebras in Z^*

4.1 Definition:

Define Symmetric Knot graphs using Knot theory. Let S_n denote a symmetric group of order n. Let $\pi \in S_n$ then π can be represented as a graph in which the vertices of π are represented in two rows such that each row contains n vertices. The vertices of each row is indexed with $1, 2, \ldots, n$ from left to right in order. Let $E(\sigma)$ denote the set of all edges of σ.

Let $E(\sigma) = \{e_i = (i, \sigma(i)); i = 1,2,3\ldots n\}$

Define A_σ such that

$A_\sigma \subseteq E(\sigma) \times E(\sigma)$ where

$A_\sigma = \{a_{ij} = (e_i, e_j); i < j, \quad e_i, e_j \in E(\sigma)\}$

$B_\sigma = \{b_{ij} = a_{ij} \in A_\sigma : \sigma(i) > \sigma(j)\}$

$S_\sigma = \{S = (s_1, s_2 \ldots s_\beta) : s_i = (1,-1)^k (or) (-1,1)^l\}$

where k and l are integers.

4.2 Knots in σ.

Let $(s_1, s_2 \ldots s_\beta) \in S_\sigma$ Then s_i is called a Knots in σ.

4.3Type I knots in σ.

If $s_i = (-1, 1)^k$ then s_i is called a Type I Knots in σ.

4.4Type II knots in σ.

If $s_i = (1, -1)^l$ then s_i is called a Type II Knots in σ.

4.5 knot in σ.

Let $s_i = (1, -1)^k \; (or) \; (1, -1)^l$ and $k = 1 \; (or) \; l = 1$ Then s_i is called a knot.

4.6 Generalized knot mapping

For each element $(s_1, s_2 \ldots s_\beta) \in S_\sigma$, we Define a mapping and which we call it as generalized knot mapping, as follows.

$f : A_\sigma \to Z^*$, where

$Z^* = \{(0,0), (1,-1)^k, (-1,1)^l\}_{k,l=1}^{\infty}$

such that $f(x) = \begin{cases} 0 & if\ x \notin B_\sigma \\ s_i & if\ x = b_i \in B_\sigma \end{cases}$

let R_σ denote the collection of all generalized knot mapping defined on σ.

4.7 Remark:

Let $R_n = \bigcup_{\sigma \in S_n} R_\sigma$.

To define multiplication in R_n, first we define the multiplication as follows.

4.8 Definition:

- $(0, 0)(0, 0) = (0, 0)$
- $(0, 0)(1, -1)^k = (1, -1)^k (0, 0) = (1,-1)^k$
- $(0, 0)(-1, 1)^l = (-1, 1)^l (0, 0) = (-1, 1)^l$
- $(1, -1)^k (-1, 1)^l = (-1, 1)^l (1, -1)^k =$

$\begin{cases} (1,-1)^k, if\ k > l \\ (-1,1)^l, if\ l > k \\ (0,0), if\ k = l \end{cases}$

Now we define multiplication among elements in R_n as follows.

4.9 Definition:

Let $f, g \in R_n$, where $f \in R_\sigma$ and $g \in R_\pi$.

Define $f.g : A_{\pi.\sigma} \to Z^*$ as follows $f.g(\alpha_i, \alpha_j) =$
$\begin{cases} f(e_i, e_j).g(\beta_i, \beta_j); & (e_i, e_j) \in B_\sigma \\ f(e_i, e_j).g(\beta_j, \beta_i); & otherwise \end{cases}$

4.10Theorem

If $a, b, c \in Z^*$, then $a(bc) = (ab)c$

Proof:

Case1: Let

$a = (1, -1)^{k_1}, b = (-1, 1)^l, c = (1, -1)^{k_2}$

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Now $(ab)=(1,-1)^{k_1}(-1,1)^{l}=$

$$\begin{cases}(1,-1)^{k_1-l} & if \quad k_1>l\\ (-1,1)^{l-k_1} & if \quad l>k_1\\ (0,0) & if \quad k_1=l\end{cases}$$

$$(ab)c=(-1,1)^{l-k_1}(1,-1)^{k_2}, \quad if \ l>k_1$$
$$=(1,-1)^{k_1+k_2-l}, \quad if \ k_1>l$$
$$(ab)c=(1,-1)^{k_1-l}(1,-1)^{k_2}, \quad if \ k_1>l$$

$(ab)c=$

$$\begin{cases}(-1,1)^{l-k_1-k_2} & if \quad l-k_1>k_2 \ and \quad l>k_1\\ (1,-1)^{k_1+k_2-l} & if \quad k_2>l-k_1 \quad and \quad l>k_1\\ (0,0) & if \quad l-k_1=k_2 \quad and \quad l>k_1\end{cases}$$

$$(ab)c=(0,0)(1,-1)^{k_2}=(1,-1)^{k_2} \quad if \ k_1=l$$

Similarly, $(bc)=(-1,\ 1)^{l}(1,-1)^{k_2}=$

$$\begin{cases}(-1,1)^{l-k_2} & if \quad l>k_2\\ (1,-1)^{k_2-l} & if \quad k_2>l\\ (0,0) & if \quad l=k_2\end{cases}$$

$$a(bc)=(1,-1)^{k_1}(1,-1)^{k_2-l}, \quad if \ k_2>l$$
$$=(1,-1)^{k_1+k_2-l}, \quad if \ k_2>l$$

And $a(bc)=(1,-1)^{k_1}(-1,\ 1)^{l-k_2}, \quad if \ l>k_2$

$$=\begin{cases}(-1,1)^{l-k_1-k_2} & if \ l-k_2>k_1 \quad and \quad l>k_2\\ (1,-1)^{k_1+k_2-l} & if \quad k_1>l-k_2 \quad and \quad l>k_2\\ (0,0) & if \quad l-k_2=k_1 \quad and \quad l>k_2\end{cases}$$

$$a(bc)=(1,-1)^{k_1}(0,\ 0)=(1,-1)^{k_1} \quad if \ l=k_2$$

Subcase1:Let

$$(ab)c=(1,-1)^{k_1+k_2-l}, \quad if \ k_1>l$$

And $a(bc)=(1,-1)^{k_1+k_2-l}, \quad if \ k_2>l$

Hence $a(bc)=(1,-1)^{k_1+k_2-l}=a(bc)$

Subcase2:Let

$$(ab)c=(-1,1)^{l-k_1-k_2} \quad if \quad l-k_1>k_2 \ and \quad l>k_1$$

And

$$a(bc)=(-1,1)^{l-k_1-k_2} \quad if \quad l-k_2>k_1 \ and \quad l>k_2$$

Hence $a(bc)=(-1,1)^{l-k_1-k_2}=(ab)c$

Sub case3:Let

$$(ab)c=(1,-1)^{k_1+k_2-l} \quad if \quad k_2>l-k_1 \ and \quad l>k_1$$

And

$$a(bc)=(1,-1)^{k_1+k_2-l} \quad if \quad k_1>l-k_2 \ and \quad l>k_2$$

Hence $a(bc)=(1,-1)^{k_1+k_2-l}=(ab)c$

Sub case4: Let

$$(ab)c=(0,0) \quad if \quad l-k_1=k_2 \quad and \quad l>k_1$$

And

$$a(bc)=(0,0) \quad if \quad l-k_2=k_1 \quad and \quad l>k_2$$

Hence $(ab)c=(0,0)=a(bc)$

Subcase5:Let $(ab)c=(1,-1)^{k_2} \quad if \ k_1=l$

And $a(bc)=(1,-1)^{k_1+k_2-l} \quad if \ k_2>l$

$$=(1,-1)^{k_2}$$

Hence $(ab)c=(1,-1)^{k_2}=a(bc)$

Case2:Let

$$a=(1,-1)^{k_1}, b=(1,-1)^{k_2}, c=(-1,1)^{l}$$
$$(ab)=(1,-1)^{k_1}(1,-1)^{k_2}=(1,-1)^{k_1+k_2}$$

Now $(ab)c=(1,-1)^{k_1+k_2}(-1,1)^{l}=$

$$\begin{cases}(-1,1)^{l-k_1-k_2} & if \quad l>k_1+k_2\\ (1,-1)^{k_1+k_2-l} & if \ k_1+k_2 \quad >l\\ (0,0) & if \quad l=k_1+k_2\end{cases}$$

Similarly, $(bc)=(1,-1)^{k_2}(-1,\ 1)^{l}=$

$$\begin{cases}(1,-1)^{k_2-l} & if \quad k_2>l\\ (-1,1)^{l-k_2} & if \quad l>k_2\\ (0,0) & if \quad l=k_2\end{cases}$$

Now $a(bc)=(1,-1)^{k_1}(1,-1)^{k_2-l}, \quad if \ k_2>l$

$$=(1,-1)^{k_1+k_2-l}, \quad if \ k_2>l$$

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And $a(bc)=(1,-1)^{k_1}(-1,\ 1)^{l-k_2},\quad if\ l>k_2$

$$=\begin{cases}(-1,1)^{l-k_1-k_2} & \text{if } l-k_2>k_1 \text{ and } l>k_2\\ (1,-1)^{k_1+k_2-l} & \text{if } k_1>l-k_2 \text{ and } l>k_2\\ (0,0) & \text{if } l-k_2=k_1 \text{ and } l>k_2\end{cases}$$

$a(bc)=(1,-1)^{k_1}(0,\ 0)=(1,-1)^{k_1}\quad if\ l=k_2$

Subcase1:Let

$a(bc)=(1,-1)^{k_1+k_2-l},\quad if\ k_2>l$

And $(ab)c=(1,-1)^{k_1+k_2-l},\quad if\ k_1+k_2>l$

Hence $a(bc)=(1,-1)^{k_1+k_2-l}=a(bc)$

Subcase2:Let

$a(bc)=(1,-1)^{k_1+k_2-l},\quad if\ k_1>l-k_2\quad and\ l>k_2$

And $(ab)c=(1,-1)^{k_1+k_2-l},\quad if\ k_1+k_2>l$

Hence $a(bc)=(1,-1)^{k_1+k_2-l}=a(bc)$

Sub Case3:Let

$(ab)c=(-1,1)^{l-k_1-k_2}\quad if\quad l-k_2>k_1\ and\quad l>k_2$

And

$a(bc)=(-1,1)^{l-k_1-k_2}\quad if\quad l>k_1+k_2$

Hence $a(bc)=(-1,1)^{l-k_1-k_2}=(ab)c$

Sub Case4:Let

$a(bc)=(0,0)\quad if\quad l-k_2=k_1\quad and\quad l>k_2$

And $(ab)c=(0,0)\quad if\quad l=k_1+k_2$

Hence $(ab)c=(0,0)=a(bc)$

Subcase5:Let $a(bc)=(1,-1)^{k_1}\quad if\ k_2=l$

And $(ab)c=(1,-1)^{k_1+k_2-l}\quad if\ k_2>l$

$=(1,-1)^{k_1}$

Hence $(ab)c=(1,-1)^{k_1}=a(bc)$

Case3:Let

$a=(1,-1)^k,\ b=(-1,1)^{l_1},\ c=(-1,1)^{l_2}$

$(ab)=(1,-1)^k(-1,1)^{l_1}=$

$$\begin{cases}(1,-1)^{k-l_1} & \text{if } k>l_1\\ (-1,1)^{l_1-k} & \text{if } l_1>k\\ (0,0) & \text{if } l_1=k\end{cases}$$

Now $(ab)c=(1,-1)^{k-l_1}(-1,1)^{l_2}$

$(ab)c=(0,0)(-1,1)^{l_2}=(-1,1)^{l_2}\ if\ k=l_1$

$(ab)c=(-1,1)^{l_1-k}(-1,1)^{l_2}=(-1,1)^{l_1+l_2-k}\quad if\quad l_1>k$

Similarly,

$(bc)=(-1,1)^{l_1}(-1,\ 1)^{l_2}=(-1,1)^{l_1+l_2}$

$$a(bc)=\begin{cases}(1,-1)^{l_1+l_2-k} & \text{if } l_1+l_2>k\\ (1,-1)^{k-l_1-l_2} & \text{if } k>l_1+l_2\\ (0,0) & \text{if } l_1+l_2=k\end{cases}$$

Subcase1: Let

$(ab)c=(1,-1)^{k-l_1-l_2},\ if\ k-l_1>l_2\ and\quad k>l_1$

And $a(bc)=(1,-1)^{k-l_1-l_2},\ if\ k>l_1+l_2$

Hence $a(bc)=(1,-1)^{k-l_1-l_2}=a(bc)$

Subcase2 :Let

$(ab)c=(-1,1)^{l_1+l_2-k},\ if\ l_2>k-l_1\quad and\ k>l_1$

And $a(bc)=(-1,1)^{l_1+l_2-k},\ if\ l_1+l_2>k$

Hence $a(bc)=(-1,1)^{l_1+l_2-k}=a(bc)$

Sub Case3: Let

$(ab)c=(0,0)\ if\quad k-l_1=l_2\ and\quad k>l_1$

And $a(bc)=(0,0)\ if\quad l_1+l_2=k$

Hence $a(bc)=(0,0)=(ab)c$

Sub Case4: Let

$(ab)c=(-1,1)^{l_1+l_2-k},\ if\ l_1>k$

And

$a(bc)=(-1,1)^{l_1+l_2-k},\ if\ l_1+l_2>k$

Hence

$(ab)c=(-1,1)^{l_1+l_2-k}=a(bc)$

Subcase5: Let $(ab)c=(-1,1)^{l_2}\quad if\ k=l_1$

And $a(bc) = (-1, 1)^{l_1 + l_2 - k} \quad if\ l_1 + l_2 > k$

$= (-1, 1)^{l_2}$

Hence $(ab)c = (-1, 1)^{l_2} = a(bc)$

Case4:

Let $a = (-1, 1)^{l_1}, b = (-1, 1)^{l_2}, c = (1, -1)^k$

$(ab) = (-1, 1)^{l_1}(-1, 1)^{l_2} = (-1, 1)^{l_1 + l_2}$

$$(ab)c = (-1, 1)^{l_1 + l_2}(1, -1)^k = \begin{cases} (1, -1)^{k - l_1 - l_2} & \text{if } k > l_1 + l_2 \\ (-1, 1)^{l_1 + l_2 - k} & \text{if } l_1 + l_2 > k \\ (0, 0) & \text{if } l_1 + l_2 = k \end{cases}$$

$$(bc) = (-1, 1)^{l_2}(1, -1)^k = \begin{cases} (1, -1)^{k - l_2} & \text{if } k > l_2 \\ (-1, 1)^{l_2 - k} & \text{if } l_2 > k \\ (0, 0) & \text{if } l_2 = k \end{cases}$$

Similarly, $a(bc) = (1, -1)^{k - l_2}(-1, 1)^{l_1}$

$$a(bc) = \begin{cases} (1, -1)^{k - l_1 - l_2} & \text{if } k > l_2 \text{ and } k - l_2 > l_1 \\ (-1, 1)^{l_1 + l_2 - k} & \text{if } k > l_2 \text{ and } l_1 > k - l_2 \\ (0, 0) & \text{if } k > l_2 \text{ and } l_1 = k - l_2 \end{cases}$$

Let

$a(bc) = (-1, 1)^{l_1}(-1, 1)^{l_2 - k}, \quad if\ l_2 > k$

$a(bc) = (-1, 1)^{l_1 + l_2 - k}, \quad if\ l_2 > k$

Let

$a(bc) = (-1, 1)^{l_1}(0, 0) = (-1, 1)^{l_1} \quad if\ l_2 = k$

Subcase1: Let

$a(bc) = (1, -1)^{k - l_1 - l_2}, \quad if\ k > l_2\ and\ k - l_2 > l_1$

$(ab)c = (1, -1)^{k - l_1 - l_2}, \quad if\ k > l_1 + l_2$

Hence $a(bc) = (1, -1)^{k - l_1 - l_2} = a(bc)$

Subcase2: Let

$a(bc) = (-1, 1)^{l_1 + l_2 - k}, \quad if\ k > l_2\ and\ l_1 > k - l_2$

And $a(bc) = (-1, 1)^{l_1 + l_2 - k}, \quad if\ l_1 + l_2 > k$

Hence $a(bc) = (-1, 1)^{l_1 + l_2 - k} = a(bc)$

Sub Case3: Let

$a(bc) = (0, 0) \quad if\ l_1 + l_2 = k$

And $(ab)c = (0, 0) \quad if\ l_1 + l_2 = k$

Hence $a(bc) = (0, 0) = (ab)c$

Sub Case4: Let

$a(bc) = (-1, 1)^{l_1 + l_2 - k}, \quad if\ l_2 > k$

And

$(ab)c = (-1, 1)^{l_1 + l_2 - k}, \quad if\ l_1 + l_2 > k$

Hence

$(ab)c = (-1, 1)^{l_1 + l_2 - k} = a(bc)$

Subcase5: Let $a(bc) = (-1, 1)^{l_1} \quad if\ k = l_2$

And $(ab)c = (-1, 1)^{l_1 + l_2 - k} \quad if\ l_1 + l_2 > k$

$= (-1, 1)^{l_1}$

Hence $(ab)c = (-1, 1)^{l_1} = a(bc)$

Case5: Let

$a = (-1, 1)^l, b = (1, -1)^{k_1}, c = (1, -1)^{k_2}$

$$(ab) = (-1, 1)^l(1, -1)^{k_1} = \begin{cases} (1, -1)^{k_1 - l} & \text{if } k_1 > l \\ (-1, 1)^{l - k_1} & \text{if } l > k_1 \\ (0, 0) & \text{if } k_1 = l \end{cases}$$

$(ab)c = (1, -1)^{k_1 - l}(1, -1)^{k_2} = (1, -1)^{k_1 + k_2 - l} \quad if \quad k_1 > l$

$(ab)c = (-1, 1)^{l - k_1}(1, -1)^{k_2} \quad if \quad l > k_1$

$$(ab)c = \begin{cases} (1, -1)^{k_1 + k_2 - l} & \text{if } l > k_1 \text{ and } k_2 > l - k_1 \\ (-1, 1)^{l - k_1 - k_2} & \text{if } l > k_1 \text{ and } l - k_1 > k_2 \\ (0, 0) & \text{if } l > k_1 \text{ and } l - k_1 = k_2 \end{cases}$$

$(ab)c = (0, 0)(1, -1)^{k_2} = (1, -1)^{k_2} \quad if \quad l = k_1$

$(bc) = (1, -1)^{k_1}(1, -1)^{k_2} = (1, -1)^{k_1 + k_2}$

$$a(bc) = (-1,\ 1)^{l}(1,-1)^{k_1+k_2}$$

$$a(bc) = \begin{cases} (1,-1)^{k_1+k_2-l} & \text{if } k_1+k_2 > l \\ (-1,1)^{l-k_1-k_2} & \text{if } l > k_1 \text{ and } l > k_1+k_2 \\ (0,0) & \text{if } l > k_1 \text{ and } l = k_1+k_2 \end{cases}$$

Subcase1 :

Let $(ab)c = (1,-1)^{k_1+k_2-l}$, $if\ k_1 > l$

$a(bc) = (1,-1)^{k_1+k_2-l}$, $if\ k_1+k_2 > l$

Hence $a(bc) = (1,-1)^{k_1+k_2-l} = a(bc)$

Subcase2: Let

$(ab)c = (1,-1)^{k_1+k_2-l}, if\ l > k_1\ and\ k_2 > l - k_1$

And $a(bc) = (1,-1)^{k_1+k_2-l}$, $if\ k_1+k_2 > l$

Hence $a(bc) = (1,-1)^{k_1+k_2-l} = a(bc)$

Sub Case3: Let

$(ab)c = (-1,1)^{l-k_1-k_2}$, $if\ l > k_1\ and\ l - k_1 > k_2$

And

$a(bc) = (-1,1)^{l-k_1-k_2}$, $if\ l > k_1\ and\ l > k_1 + k_2$

Hence

$(ab)c = (-1,1)^{l-k_1-k_2} = a(bc)$

Sub Case4: Let

$(ab)c = (0,0)\ \ if\ \ l - k_1 = k_2\ \ and\ \ l > k_1$

And

$a(bc) = (0,0)\ \ if\ \ l = k_1 + k_2\ \ and\ \ l > k_1$

Hence $a(bc) = (0,0) = (ab)c$

Subcase5: Let $(ab)c = (1,-1)^{k_2}\ \ if\ l = k_1$

And $a(bc) = (1,-1)^{k_1+k_2-l}\ \ if\ k_1+k_2 > l$

$= (1,-1)^{k_2}$

Hence $(ab)c = (1,-1)^{k_2} = a(bc)$

Case6: Let

$a = (-1,1)^{l_1}, b = (1,-1)^{k}, c = (-1,1)^{l_2}$

$(ab) = (-1,1)^{l_1}(1,-1)^{k} =$

$$\begin{cases} (1,-1)^{k-l_1} & \text{if } k > l_1 \\ (-1,1)^{l_1-k} & \text{if } l_1 > k \\ (0,0) & \text{if } l_1 = k \end{cases}$$

$(ab)c = (-1,1)^{l_1-k}(-1,1)^{l_2} = (-1,1)^{l_1+l_2-k}\ \ if\ \ l_1 > k$

$(ab)c = (1,-1)^{k-l_1}(-1,1)^{l_2}\ \ if\ \ k > l_1$

$$(ab)c = \begin{cases} (1,-1)^{k-l_1-l_2} & \text{if } k > l_1 \text{ and } k-l_1 > l_2 \\ (-1,1)^{l_1+l_2-k} & \text{if } k > l_1 \text{ and } l_2 > k-l_1 \\ (0,0) & \text{if } k > l_1 \text{ and } k-l_1 = l_2 \end{cases}$$

$(ab)c = (0,0)(-1,1)^{l_2} = (-1,1)^{l_2}\ \ if\ \ l_1 = k$

$(bc) = (1,\ -1)^{k}(-1,1)^{l_2} =$

$$\begin{cases} (1,-1)^{k-l_2} & \text{if } k > l_2 \\ (-1,1)^{l_2-k} & \text{if } l_2 > k \\ (0,0) & \text{if } l_2 = k \end{cases}$$

$a(bc) = (-1,1)^{l_1}(1,-1)^{k-l_2}\ \ if\ \ k > l_2$

$$a(bc) = \begin{cases} (1,-1)^{k-l_1-l_2} & \text{if } k > l_2 \text{ and } k-l_2 > l_1 \\ (-1,1)^{l_1+l_2-k} & \text{if } k > l_2 \text{ and } l_1 > k-l_2 \\ (0,0) & \text{if } k > l_2 \text{ and } l_1 = k_1 - l_2 \end{cases}$$

$a(bc) = (-1,1)^{l_1}(-1,1)^{l_2-k} = (-1,1)^{l_1+l_2-k}\ \ if\ \ l_2 > k$

$a(bc) = (-1,1)^{l_1}(0,0) = (-1,1)^{l_1}\ \ if\ \ l_2 = k$

Subcase1:Let

$a(bc) = (1,-1)^{k-l_1-l_2}$, $if\ k > l_2\ \ and\ \ k - l_2 > l_1$

$(ab)c = (1,-1)^{k-l_1-l_2}$, $if\ k > l_1\ \ and\ \ l_2 > k - l_1$

Hence $a(bc) = (1,-1)^{k-l_1-l_2} = a(bc)$

Subcase2: Let

$a(bc) = (-1,1)^{l_1+l_2-k}, if\ \ k > l_2\ \ and\ l_1 > k - l_2$

And

$(ab)c = (-1,1)^{l_1+l_2-k}$, $if \quad l_2 > k - l_1 \quad and \quad k > l_1$

Hence $a(bc) = (-1,1)^{l_1+l_2-k} = a(bc)$

Sub Case3: Let

$a(bc) = (0,0) \quad if \quad l_1 = k - l_2 \quad and \quad k > l_2$

And

$(ab)c = (0,0) \quad if \quad k - l_1 = l_2 \quad and \quad k > l_1$

Hence $a(bc) = (0,0) \quad = (ab)c$

Sub Case4:

Let $a(bc) = (-1,1)^{l_1+l_2-k}, \quad if \ l_2 > k$

And $(ab)c = (-1,1)^{l_1+l_2-k}, \quad if \ l_1 > k$

Hence

$(ab)c = (-1,1)^{l_1+l_2-k} = a(bc)$

Subcase5: Let $a(bc) = (-1,1)^{l_1} \quad if \ l_2 = k$

And

$(ab)c = (-1,1)^{l_1+l_2-k} \quad if \ l_1 > k$

$= (-1,1)^{l_1}$

Hence $(ab)c = (-1,1)^{l_1} = a(bc)$

4.11 Theorem:

If $f, g, h \in R_n$, then $(fg)h = f(gh)$

Proof:

Let $f \in R_\sigma, g \in R_\pi \ \& \ h \in R_\delta$,

where $\sigma, \pi, \delta \in S_n$

The above multiplication Z^* is associative.

Hence R_n is associative.

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